Abstracts for PANTS XXXIX

The local Langlands program and cuspidal vector bundles

Alexander Bertoloni Meli, Boston University

Abstract: I will motivate the local Langlands program for p-adic groups and discuss recent categorical enhancements stemming from the work of Fargues and Scholze. I will discuss work in progress with Teruhisa Koshikawa on understanding supercuspidal representations on the Galois side of the correspondence.

An explicit zero-free region for automorphic L-functions

Steven Creech, Brown University

Abstract: The Riemann hypothesis says that all zeros of $\zeta(s)$ lie on the line $\mathrm{Re}(s) = \frac{1}{2}$. While the Riemann hypothesis is far from being solved, one could ask if there is some region where we know that there are no zeros, such regions are called zero-free regions. In this talk, I will talk about the analogous problem for automorphic L-functions, and describe two tricks that can be done to improve explicit zero-free regions. Namely, I shall discuss a Stetchkin's trick and the use of higher degree polynomials. This is joint work with Alia Hamieh, Simran Khunger, Kaneenika Sinha, Jakob Streipel, and Kin Ming Tsang.

Primes with small primitive roots

Mikhail Gabdullin, University of Illinois Urbana-Champaign

Abstract: Let $\delta(p)$ tend to zero arbitrarily slowly as $p \to \infty$. We exhibit an explicit set $\mathcal{S}$ of primes p, defined in terms of simple functions of the prime factors of $p-1$, for which the least primitive root of p is at most $p^{1/4-\delta(p)}$ for all $p \in \mathcal{S}$, where $\#\{p \leq x : p \in \mathcal{S}\} \sim \pi(x)$ as $x \to \infty$. This is a joint work with Kevin Ford and Andrew Granville.

Hecke-towers and modular product formulas

Michael Griffin, Vanderbilt University

Abstract: There are three important product formulas for expressions of the type

$$\prod \big(j(\sigma) - j(z)\big).$$

Here j is the modular j-invariant which distinguishes isomorphism classes of elliptic curves and generates the field of modular functions for $\mathrm{SL}_2(\mathbb{Z})$, and z and σ are each either complex variables, or run over a complete set of representatives of imaginary quadratic numbers of fixed discriminant. These three identities are:

1) The denominator formula for the Monster Lie-algebra,
2) Borcherd's product formulas for the Hilbert class functions,
3) The Gross–Zagier formula for norms of singular moduli.

Borcherds notes that, despite the similarity of the "left hand sides" of these identities, their proofs are wildly different, and "there does not seem to be any obvious way to deduce any of these 3 formulas from the others." Motivated by this statement, we will show how these three identities can be derived

from a cohesive theory, each identity building from the previous, and rooted in the algebraic structure of spaces of modular objects.

A coupling for prime factors

Tony Haddad, Université de Montréal

Abstract: We present a coupling between a random integer N_x, chosen uniformly from the interval $[1, x]$, and a Poisson-Dirichlet process $(V_i)_{i\geq 1}$ satisfying

$$\mathbb{E} \sum_{i\geq 1} |\log P_i - V_i \log x| \asymp 1,$$

where $N_x = P_1 P_2 \cdots$ is the unique factorization of N_x into primes (or ones), and the P_i's are non-increasing. This resolves a conjecture posed by Arratia in 1998. We also explain how to apply the coupling to extract statistical information about divisors. This is joint work with Dimitris Koukoulopoulos.

On covering systems and related problems

Nicholas Jones, College of Charleston

Abstract: A covering system $\mathcal{C}$ is a finite set of congruences such that the union of their equivalence classes cover the set of integers. In a paper published in 1950, Erdös introduced the concept of covering systems. This talk will serve as a current exposition to the topic of covering systems intended for those who are seeking a review of the topic. We will explore the proposed problems from Erdös's paper and their progress in the literature such as that of the minimum modulus problem and the odd covering problem. If time permits, I will discuss progress I have made on the non-existence of a distinct covering system in the interval $[n, 10n]$ for $n \geq 3$.

On extreme values of $r_3(n)$ in arithmetic progressions

Jonah Klein, University of South Carolina

Abstract: Let $r_3(n)$ denote the number of ways of writing an integer n as a sum of three integer squares. Based on an idea of Chowla, we show that for any integer m, and any residue $a \mod m$ for which a can be written as a sum of three squares modulo m, there are infinitely many integers $n \equiv a \mod m$ such that $r_3(n) \gg_m \sqrt{n} \log\log(n)$. This is work in progress with Michael Filaseta and Cihan Sabuncu.

On the distribution of class groups — beyond Cohen-Lenstra and Gerth

Yuan Liu, University of Illinois Urbana-Champaign

Abstract: The Cohen-Lenstra heuristic studies the distribution of the p-part of class group of quadratic number fields for odd prime p, and Gerth's conjecture regards the distribution of the 2-part of class group of quadratic fields. The main difference between these two conjectures is that while (odd) p-part of class group behaves completely "randomly", the 2-part of class group does not since the 2-torsion of class group is controlled by the genus field. In this talk, we will discuss a new conjecture generalizing Cohen-Lenstra and Gerth's conjectures. The techniques involves Galois cohomology and embedding problem of global fields. If time permits, we will also discuss how to prove a function field analog of this new conjecture, by counting points on the Hurwitz spaces.

Shimura lift of Rankin-Cohen brackets of Eisenstein series and theta function

Tianyu Ni, Clemson University

Abstract: Let $\theta(z) = \sum_{n\in\mathbb{Z}} q^{n^2} \in M_{1/2}(4)$ be the Jacobi theta function. Selberg observed that for a normalized Hecke eigenform $f(z) \in M_\ell(1)$ with $a_f(1) = 1$, the first Shimura lift provides the identity $\mathcal{S}_1(f(4z)\theta(z)) = f(z)^2 \in M_{2\ell}(1)$. In this talk, we will talk about some generalizations of Selberg's identity and its application to the non-vanishing of (twisted) central L-values.

How large are the eigenvalues of the Hecke operators?

Erick Ross, Clemson University

Abstract: In this presentation, we determine the average size of the eigenvalues of the Hecke operators in both the vertical and the horizontal perspective. The "average size" is measured via the quadratic mean.

Root number correlation bias of Fourier coefficients of modular forms

Nina Zubrilina, Massachusetts Institute of Technology

Abstract: In a recent study, He, Lee, Oliver, and Pozdnyakov observed a striking oscillating pattern in the average value of the P-th Frobenius trace of elliptic curves of prescribed rank and conductor in an interval range. Sutherland discovered that this bias extends to Dirichlet coefficients of a much broader class of arithmetic L-functions when split by root number. In my talk, I will discuss this root number correlation in families of holomorphic and Maass forms.
